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الأساتذة

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محاضرة 4

Bode diagram :-

Stability

① Absolute Stability

- Routh.

② Relative stability

- Bode Diagram.

- polar plot..

- Nyquist.

in relative stability you know to what range or to what degree the system is stable through stability indicators (phase margin (PM) and gain margin (gm))

الكل؛ اذات ال phase margin وال gain margin
كلما زادت درجة استقرار النظام

* Frequency Analysis

given T.F. in s-domain

and we want to describe it in freq. domain

we replace $s \rightarrow j\omega$ # given O.L.T.F $G H(s)$ to get Bode Diagram

$$① s \rightarrow j\omega \Rightarrow G H(j\omega) = G H(s) \Big|_{s=j\omega}$$

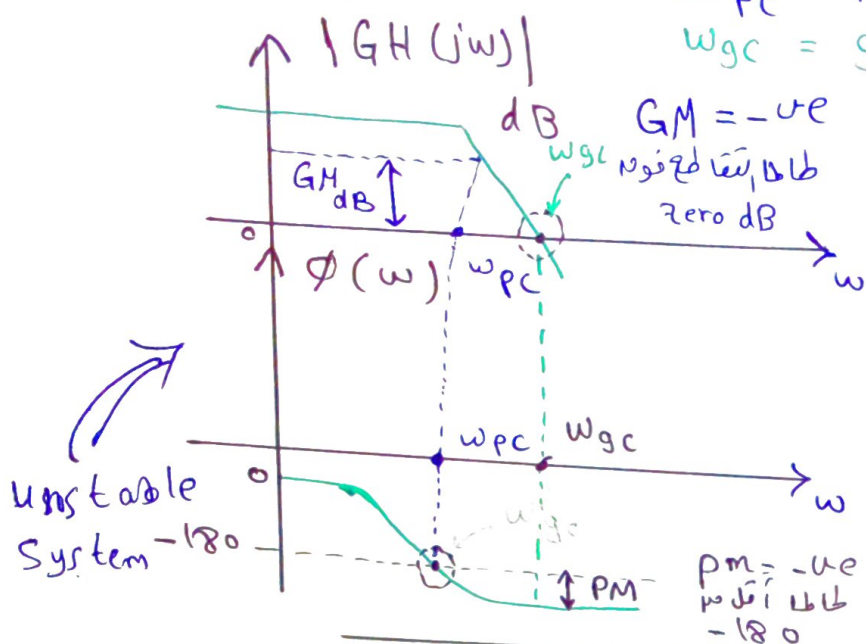
$$② |G H(j\omega)| \text{ magnitude } \angle G H(j\omega) \text{ phase}$$

$$③ |G H(j\omega)|_{dB} = 20 \log |G H(j\omega)|$$

 \Rightarrow Turn over

④ Diagram

ω_{pc} = phase cross freq.
 ω_{gc} = gain cross freq.



المعلومة ال magnitude
 وال phase تقدر
 تحديد Stability
 حالة زاي

$$| | = \sqrt{\text{Real}^2 + \text{img}^2}$$

$$\phi = \tan^{-1} \left(\frac{\text{img}}{\text{Real}} \right)$$

$$| |_{dB} = 0$$

$$\angle GH(j\omega) = -180$$

ط أدرس ال system

$$\text{Ch. eq} \Rightarrow 1 + GH(s) = 0$$

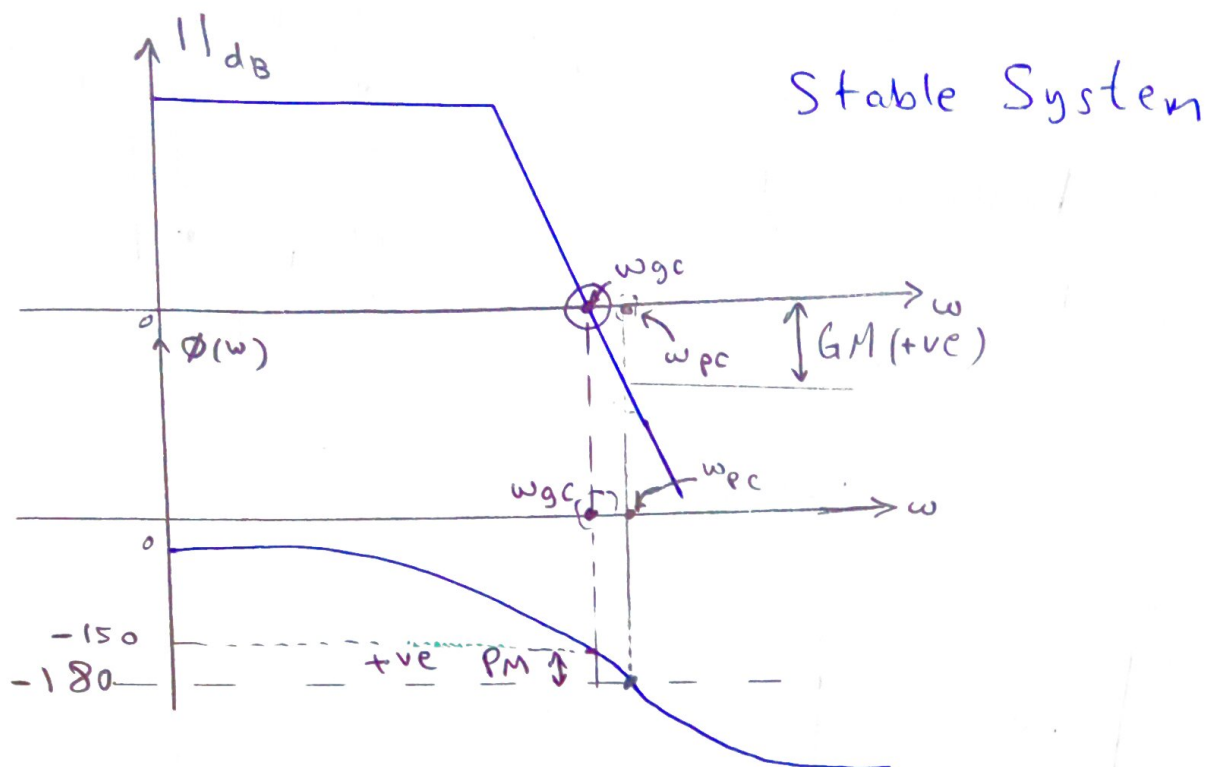
$$GH(s) = -1 + j0$$

Stability:-

- ① $GM \& PM > 0$ (+ve) \Rightarrow Stable
- ② $GM \& PM < 0$ (-ve) \Rightarrow unstable
- ③ $GM_{dB} \& PM = 0 \Rightarrow$ critically stable

Stability زادت GM & PM

\Rightarrow Turn over



$$PM = 180 + \phi(\omega) \quad | \quad \omega = \omega_{gc}$$

* The open loop T.F. $GH(s)$ can be in the following forms:

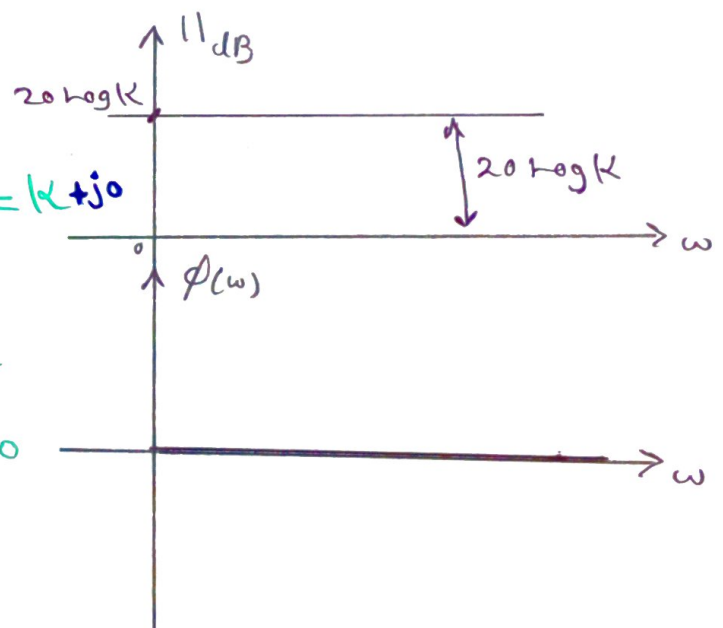
① $GH(s) = K$

① $s \rightarrow j\omega \Rightarrow GH(j\omega) = K + j0$

② $|GH(j\omega)| = K$

③ $|GH(j\omega)|_{dB} = 20 \log K$

④ $\phi(\omega) = \tan^{-1}\left(\frac{0}{K}\right) = 0$

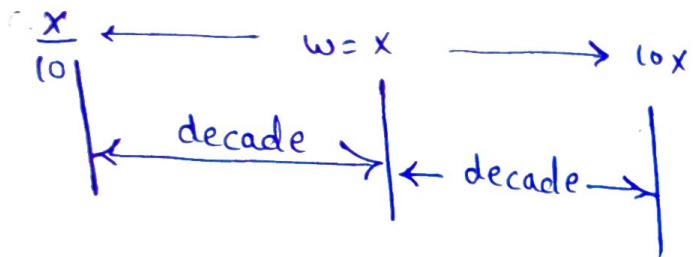
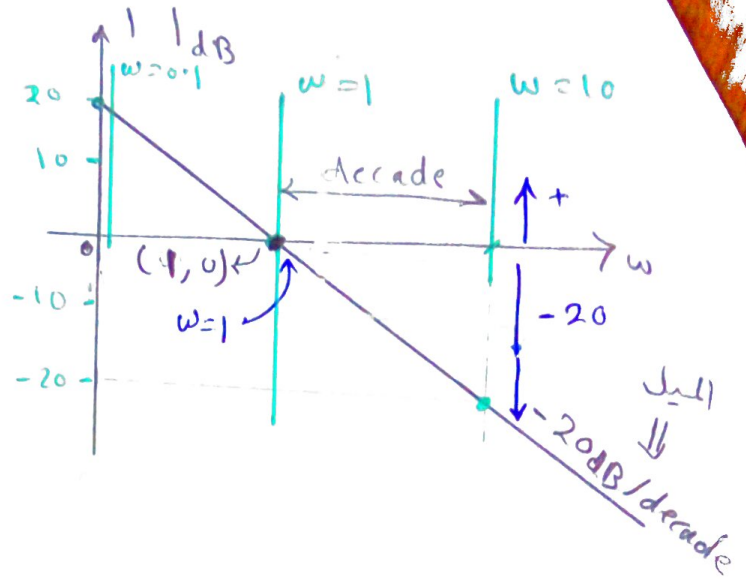


② $G_H(s) = \frac{1}{s}$

$$\textcircled{1} \quad s \rightarrow j\omega \Rightarrow GH(j\omega) = \frac{1}{j\omega}$$

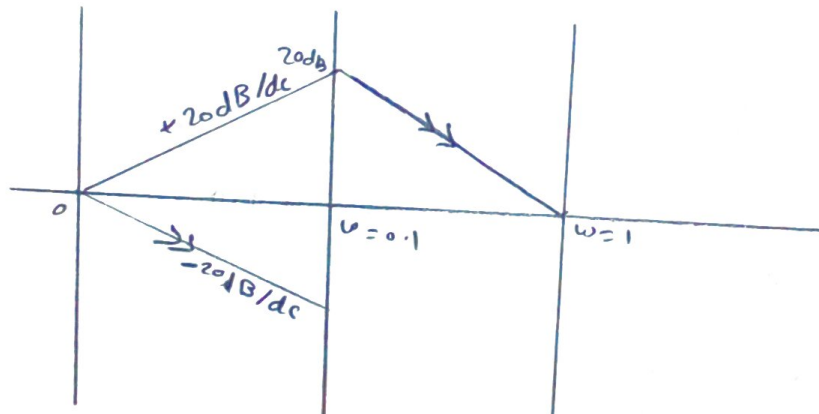
$$(2) |GH(j\omega)| = \frac{1}{\omega}$$

$$\begin{aligned} \textcircled{3} \quad |GH(j\omega)| &= 20 \log\left(\frac{1}{\omega}\right) \\ &\text{dB} \quad \swarrow \quad \searrow \\ &= 20 \log \omega^{-1} \\ &= -20 \log \omega \end{aligned}$$



ω	0.1	1	10	100
	1 dB	20 dB	0	-40 dB

طریقہٴ اُفری لرسم پیدا ہوازی



③ $GH(s) = \frac{1}{s^2}$

$$\textcircled{1} s \rightarrow \omega \Rightarrow GH(\omega) = \frac{1}{(\omega)^2}$$

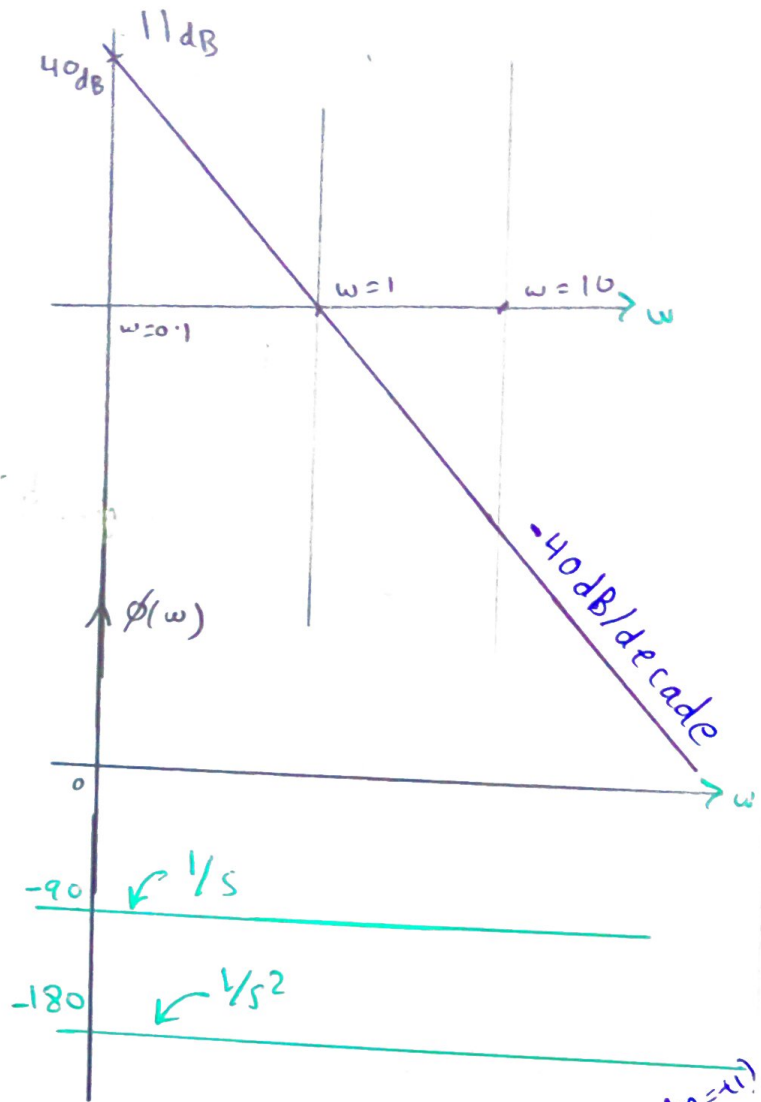
$$\textcircled{2} |GH(j\omega)| = \frac{1}{\omega^2}$$

$$\begin{aligned} \textcircled{3} \quad | |_{dB} &= 20 \log \left(\frac{1}{\omega^2} \right) \\ &= 20 \log \omega^{-2} \\ &= -40 \log \omega \end{aligned}$$

يُحْكَمُ الرُّوحُ بِنَفْسِ الطَّرِيقَتَيْنِ السَّائِقَتَيْنِ
فِي الْمَقَالِ السَّابِقِ

$$\frac{1}{s} \Rightarrow \phi(\omega) = -90^\circ$$

$$\frac{1}{s^2} \Rightarrow \phi(\omega) = -180^\circ$$

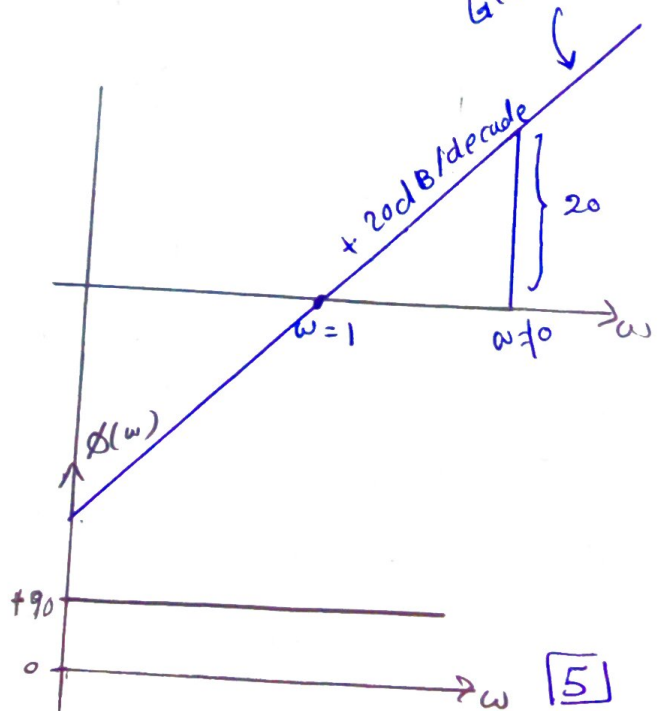


④ $G_H(s) = (s)^{\pm n}$

$\omega = 1$ فصلتیم میر
 \Rightarrow $\frac{1}{dB}$ اصلی
 $+20n \text{ dB/decade}$ بسط
بسط

$$\phi(\omega) = \pm 90^\circ$$

⇒ turn over



$$⑤ \quad G H(s) = \frac{K}{s}$$

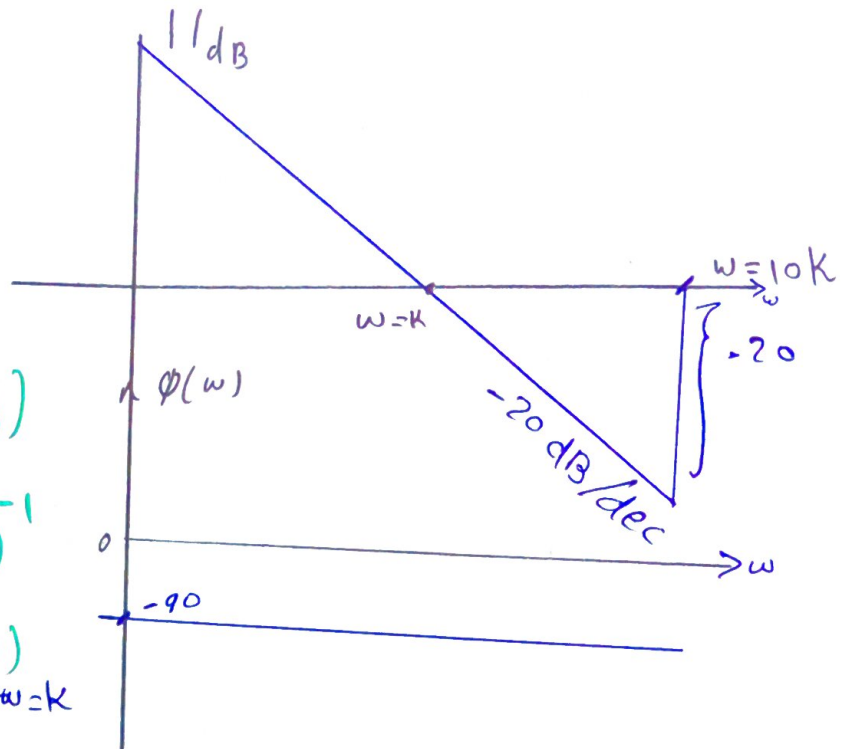
$$- s \rightarrow j\omega \Rightarrow G H(j\omega) = \frac{K}{j\omega}$$

$$|G H(j\omega)| = \frac{K}{\omega}$$

$$|G H(j\omega)|_{dB} = 20 \log\left(\frac{K}{\omega}\right)$$

$$= 20 \log\left(\frac{K}{\omega}\right)^{-1}$$

$$= -20 \log\left(\frac{\omega}{K}\right) \quad \text{at } \omega = K$$



$$⑥ \quad G H(s) = \frac{K}{s^2}$$

خط مستقيم يمر بـ $\omega = \sqrt{K}$ والميل -40 dB/decade

وزاوية $\phi(\omega) = -180$

$$⑦ \quad G H(s) = \frac{K}{s^3}$$

خط مستقيم يمر بـ $\omega = \sqrt[3]{K}$ والميل -60 dB/decade

وزاوية $\phi(\omega) = -270$

$$⑧ \quad G H(s) = \left(1 + \frac{s}{c}\right)$$

$$- s \rightarrow j\omega$$

$$\Rightarrow G H(j\omega) = \left(1 + j\frac{\omega}{c}\right)$$

$$- |G H(j\omega)| = \sqrt{1 + \left(\frac{\omega}{c}\right)^2}$$

$$- \phi(\omega) = \tan^{-1}\left(\frac{\omega}{c}\right)$$

$$|G H(j\omega)|_{dB} = 20 \log \sqrt{1 + \left(\frac{\omega}{c}\right)^2}$$

صغيرة على ω

صغيرة عند $\omega > c$

صغيرة عند $\omega < c$

approximation

① $\omega \ll c \Rightarrow \left(\frac{\omega}{c}\right)^2 \ll 1$

$$|GH(j\omega)|_{dB} = 20 \log \sqrt{1+0} = 0 \text{ dB}$$

② $\omega > c \Rightarrow \left(\frac{\omega}{c}\right)^2 \gg 1$

$$|GH(j\omega)|_{dB} = 20 \log \sqrt{0 + \left(\frac{\omega}{c}\right)^2} = 20 \log \left(\frac{\omega}{c}\right)$$

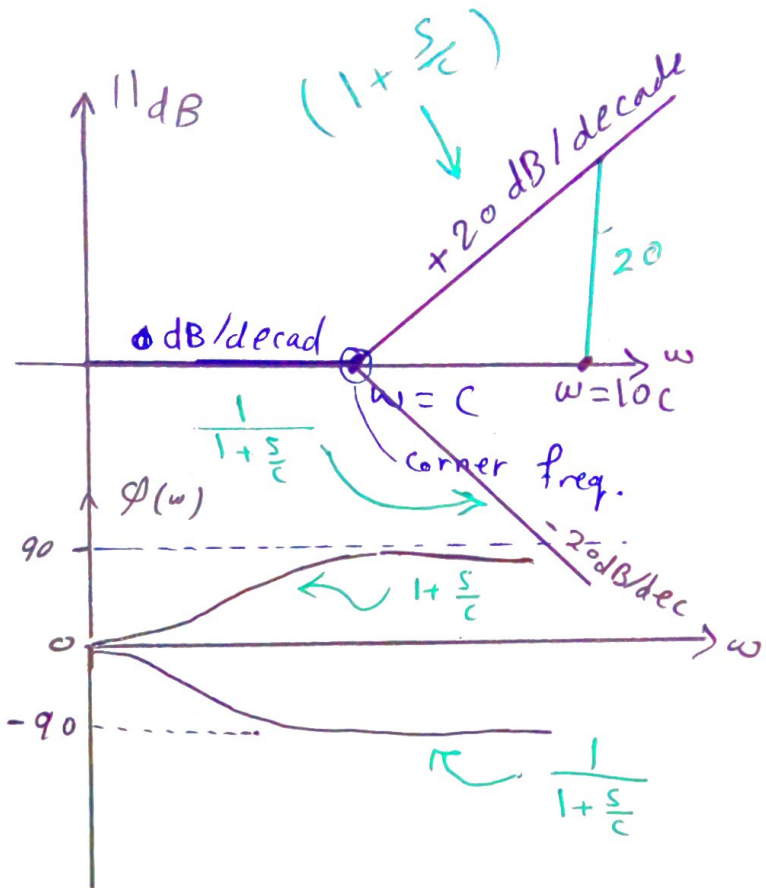
$\phi(\omega) \Rightarrow 0 \rightarrow 90$

where $\phi(\omega) = \tan^{-1}\left(\frac{\omega}{c}\right)$

ω	0		∞
$\phi(\omega)$	0		+90

For $GH(s) = \frac{1}{(1 + \frac{s}{c})}$

نفس اللي كان مع الميل (سالب) وسالب
انزاوية



* $GH(s) = \left(1 + \frac{s}{c}\right)^{\pm n}$

C = corner freq.

0 dB/dec = الميل قبل $\omega = c$

$\pm 20 \text{ dB/dec} =$ بعد $\omega = c$

$\phi(\omega) = \pm n \tan^{-1}\left(\frac{\omega}{c}\right)$

$\omega = 0 \Rightarrow 0$
انزاوية
 $\omega = \infty \Rightarrow \pm n \neq 90$
بينهم

(continued)

Term	$\phi(\omega)$	$ G _{dB}$
K	0	
$\frac{1}{s} \rightarrow \frac{1}{j\omega}$	-90	
$s \rightarrow j\omega$	+90	
$\frac{1}{s^2} \Rightarrow \frac{1}{j\omega \cdot j\omega}$	-180	
$\frac{K}{s} \rightarrow \frac{K}{j\omega}$	-90	
$\frac{K}{s^2} \rightarrow \frac{K}{j\omega \cdot j\omega}$	-180	
$1 + \frac{s}{c} \Rightarrow 1 + j\frac{\omega}{c}$	$\tan^{-1}\left(\frac{\omega}{c}\right)$	
$\frac{1}{1 + \frac{s}{c}} \Rightarrow \frac{1}{1 + j\frac{\omega}{c}}$	$\tan^{-1}\left(\frac{\omega}{c}\right)$	

Ex: $G H(s) = \frac{10}{(1+s)(1+0.1s)}$

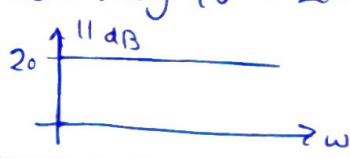
Draw the Bode Diagram and find GM & PM

$s \rightarrow j\omega$

$$G H(j\omega) = \frac{10}{(1+j\omega)(1+0.1j\omega)} = \frac{10}{(1+j\frac{\omega}{1})(1+j\frac{\omega}{10})}$$

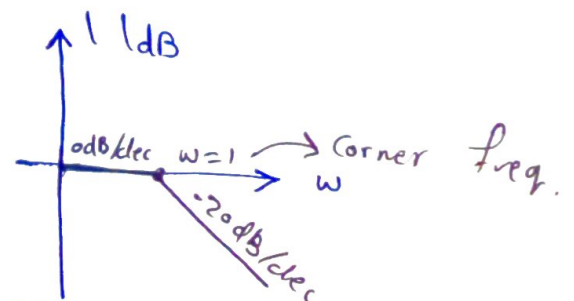
$$\phi(\omega) = 0 - \tan^{-1}(\omega) - \tan^{-1}(\frac{\omega}{10})$$

\uparrow \uparrow \uparrow
 $\frac{1}{1+j\omega}$ $\frac{1}{1+j\frac{\omega}{10}}$

term	$\phi(\omega)$	11 dB
10	0	$20 \log 10 = 20$ 

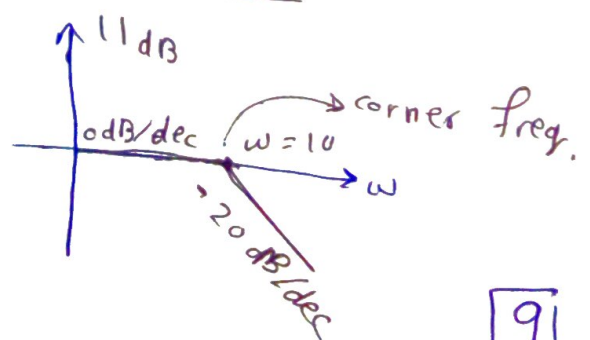
$$\frac{1}{1+\frac{s}{1}} \Rightarrow \frac{1}{1+j\frac{\omega}{1}}$$

$$-\tan^{-1}(\omega)$$



$$\frac{1}{1+\frac{s}{10}} \Rightarrow \frac{1}{1+j\frac{\omega}{10}}$$

$$-\tan^{-1}(\frac{\omega}{10})$$



ω	0	0.1	1	10	100	∞
$\phi(\omega)$	0	-6.3	-50.7	-129.3	-173.7°	-180



فردی لرحم ال phase

$$PM = 180 + \phi(\omega = \omega_{gc})$$

$$= 180 - 129.3 = + \checkmark$$

$$GM = \infty$$

\Rightarrow Stable System

